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A NEURAL NETWORK-BASED ALGORITHM FOR PRICING EUROPEAN BARRIER OPTIONS

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Abstract. In recent decades, researchers in various disciplines have made many efforts to find an appropriate approach to predict the financial market. Since it is very difficult to fully predict the financial markets, machine learning methods can help to improve this prediction. The neural network method is one of these machine learning methods that works better in modeling economic behaviors. In this study, the neural network algorithm based on an improved extreme learning machine with Legendre polynomial bases is used to determine the price of the double-barrier option. The Kronecker product of two Legendre polynomials is chosen as the basis functions of the hidden neurons. The model we consider to evaluate the double barrier option is a generalization of the Cox model, which represents the inverse relationship between the price of the underlying asset and its volatility. In this model, the parameters of the problem are chosen as time-dependent functions. First, we implement the neural network method on the proposed model and then show the correctness of the method by presenting several numerical results.

1. Introduction

The huge profits generated by stock options trading over the past decade have made this financial instrument extremely popular in the securities markets, attracting many investors and speculators. An option is a right, not an obligation, to exercise a specific underlying asset at an agreed strike price on or before the expiration date. Option holders have the right to exercise their option or not, depending on market conditions. However, if option holders fail to manage this right properly, their investment portfolio

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may be severely jeopardized. In financial markets, options are used by investors to control investment risks.

In financial markets, there are various types of options, including: European, American, Asian, barrier, lookback, etc. In this study, among the above types of options, we intend to focus on the barrier option.

The option pricing model plays an important role in modern financial theories, as it has been used in cost budgets [6, 13, 41], asset valuations [40], resource allocation [20], etc. The study of option pricing and its modeling problem is of great importance. The first mathematical studies of financial markets date back to 1900 [48]. In 1973, the classic Black-Scholes model was proposed by Fisher Black and Myron Scholes and became the benchmark for fair market option pricing. Over the decades, the Black-Scholes model has been widely accepted because it can effectively model the options pricing and provides a mechanism for extracting implied volatility. It is also used in the price of commodity futures and investment portfolios. In an ideal market, a European option price can be obtained by solving the partial differential equation (classical Black-Scholes model)

$$\frac{\partial V(S,t)}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2V(S,t)}{\partial S^2}+rS\frac{\partial V(S,t)}{\partial S}-rV(S,t)=0,\quad (S,t)\in(0,+\infty)\times(0,T), \tag{1.1}$$

under the final and boundary conditions

$$\text{Call option}:\begin{cases} V(S,T)=\max(S-K,0), \quad 0<S<+\infty,\\ V(0,t)=0, \quad \lim_{S\to+\infty}V(S,t)=S-Ke^{-r(T-t)}, \quad 0<t<T,\end{cases}$$

$$\text{Put option}:\begin{cases} V(S,T)=\max(K-S,0), \quad 0<S<+\infty,\\ V(0,t)=Ke^{-r(T-t)}, \quad \lim_{S\to+\infty}V(S,t)=0, \quad 0<t<T,\end{cases}$$

where S is underlying asset price, σ is annual volatility of asset price, r is risk-free interest rate, K is striking price, T is expiration date, and t is the elapsed time of the option's life. In the above equation, the price of the underlying asset S follows the geometric Brownian motion and satisfies in the following stochastic differential equation

$$\mathrm{d}S_t=\mu S_t\mathrm{d}t+\sigma S_t\mathrm{d}W,$$

where $\mu S_t\mathrm{d}t$ is drift, μ is mean growth rate, $\sigma S_t\mathrm{d}W$ is diffusion, and W is a geometric Brownian motion process in probability space.

For decades, the classical Black-Scholes model has been used to price options, commodity futures, and investments. However, this model has some shortcomings, including:

- No dividends are allocated to stocks.
- The model parameters are assumed to be fixed.
- Transactions are free of any fees or taxes.
- They are performed continuously.

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- This model does not show the inverse relationship between the price of the underlying asset and its volatility that is observed in market reality.

To overcome the shortcomings of the classical Black-Scholes model, extended models such as Cox models have been introduced [15]. Models developed from the classic Black-Scholes model do not have a closed-form solution due to their high computational complexity. Therefore, numerical methods are better alternatives for solving such mathematical models. Approach for option pricing are classified into two categories of parametric and nonparametric methods.

Parametric models are usually described by a smaller number of parameters, which can simplify the analysis. However, nonparametric methods may require more calculations or be more difficult to interpret the results due to the internal layers that are created. On the other hand, we know that financial markets behave nonlinearly, and this nonlinear learning capability allows nonparametric methods to model these behaviors well. While parametric methods usually depend on linear assumptions. Therefore, nonparametric methods are often preferred in financial problems due to their very high learning capabilities, flexibility, and adaptability to complex and large data.

In recent years, many researchers have turned to nonparametric methods such as neural networks and support vector regression (SVR) methods for option pricing [7, 23, 21, 31, 51]. Neural network methods are a subset of machine learning algorithms because they allow systems to learn from data and improve without being explicitly programmed.

In the following, our goal is to apply the aforementioned Legendre neural network method to price the European double barrier knock-out option under the Cox model, which is known as the constant elasticity of variance (CEV) model. Cox introduced this model to show the inverse relationship between the price of the underlying asset and its volatility. If in the model (1.1), we consider the volatility as $\sigma(S) = \delta S^{\beta}$, we obtain the following equation [14]:

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\delta^2 S^{2\beta+2}\frac{\partial^2 V(S,t)}{\partial S^2} + rS\frac{\partial V(S,t)}{\partial S} - rV(S,t) = 0, \quad B_l < S < B_u, \quad 0 < t < T, \tag{1.2}$$

where $\sigma(S)$ is the local volatility function, β is the elasticity of volatility, δ is the scale parameter of the initial instantaneous volatility at time $t = 0$, and $\sigma_0 = \sigma(S_0) = \delta S_0^{\beta}$. If β is set to zero in the differential equation (1.2), then the Cox model will be the classic Black-Scholes model. Since in this paper we seek to consider a more efficient model that is closer to the real market model, we adopt a generalization of the Cox model. In this model, the dividend yield, which we denote by q, is attributed to the underlying asset and the parameters of interest rate and dividend yield are functions of time [44]:

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\delta^2 S^{2\beta+2}\frac{\partial^2 V(S,t)}{\partial S^2} + (r(t) - q(t))S\frac{\partial V(S,t)}{\partial S} - r(t)V(S,t) = 0, \tag{1.3}$$

with the final and boundary conditions:

$$\text{Call option}: \begin{cases} V(S,T) = \max(S-K,0), \quad B_l < S < B_u, \\ V(B_l,t) = \Psi(t), \quad V(B_u,t) = \Phi(t), \quad 0 < t < T, \end{cases} \tag{1.4}$$

$$\text{Put option}: \begin{cases} V(S,T) = \max(K-S,0), \quad B_l < S < B_u, \\ V(B_l,t) = \Psi(t), \quad V(B_u,t) = \Phi(t), \quad 0 < t < T, \end{cases} \tag{1.5}$$

where B_l and B_u are the lower and upper bound barriers, respectively. The functions $\Psi(t)$ and $\Phi(t)$ are the discounts paid when the price of the underlying asset hits the respective barrier levels. This generalized Black-Scholes equation will be used to price the barrier option.

2. Main Results

Theorem 1. *Suppose the vector $P(S)$ is defined as $P(S) = [P_0(S), P_1(S), \cdots, P_{N+1}(S)]^T$ where $P_n(S)$, $n = 0, 1, \cdots, N+1$, is the nth shifted Legendre polynomial in the interval $[0,1]$ and the vector $P^{'}(S)$ is defined as $P^{'}(S) = [P_0^{'}(S), P_1^{'}(S), \cdots, P_{N+1}^{'}(S)]^T$ where $P_n^{'}(S)$, $n = 0, 1, \cdots, N+1$ is the first-order derivative of the shifted Legendre polynomial $P_n(S)$. Also, using the properties of the derivative of the shifted Legendre polynomial, we have:*

$$P_{n+1}^{'}(S) = P_{n-1}^{'}(S) + 2(2n+1)P_n(S).$$

Therefore, with the help of the above recursion relation, we will have the matrix form $P^{'}(S) = MP(S)$ where the matrix M is defined as follows:

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 2 & 0 & 10 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 6 & 0 & 14 & 0 & \cdots & 0 & 0 \\ 2 & 0 & 10 & 0 & 18 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 6 & 0 & 14 & 0 & \cdots & 2(2N+1) & 0 \\ 2 & 0 & 10 & 0 & 18 & \cdots & 0 & 0 \end{bmatrix}_{(N+2)\times(N+2)}.$$

From the Kronecker product of two Legendre polynomials $P(S) = [P_0(S), P_1(S), \cdots, P_{N+1}(S)]^T$ and $P(t) = [P_0(t), P_1(t), \cdots, P_{N+1}(t)]^T$, we get:

$$\begin{aligned}\zeta(S,t) = P(S) \otimes P(t) =& [P_0(S)P_0(t), P_0(S)P_1(t), \cdots, P_0(S)P_{N+1}(t) \\ & P_1(S)P_0(t), P_1(S)P_1(t), \cdots, P_1(S)P_{N+1}(t), \cdots, \\ & P_{N+1}(S)P_0(t), P_{N+1}(S)P_1(t), \cdots, P_{N+1}(S)P_{N+1}(t)]^T.\end{aligned}$$

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$\zeta(S,t)$ is the basis function of the hidden neural networks.

Now, the theory of the European double barrier knock-out option pricing is described using the neural network method. Note that the valuation process of the double-barrier put option is the same as the double-barrier call option.

Theorem 2. *Consider the continuous function of two variables $V(S,t) : [B_l, B_u] \times [0,T] \to \mathbb{R}$. For $N \in \mathbb{N}$, constant values γ_{ij}, $i,j = 0,1,\cdots,N+1$, and the shifted Legendre polynomials $P_0(S), P_1(S), \cdots, P_{N+1}(S)$ and $P_0(t), P_1(t), \cdots, P_{N+1}(t)$, exist such that the shifted Legendre neural network with $(N+1)^2$ neuron will be as follows:*

$$\tilde{V}(S,t) = \sum_{i=0}^{N+1}\sum_{j=0}^{N+1} \gamma_{ij} P_i(S) P_j(t) = \zeta^T(S,t)\Gamma, \tag{2.1}$$

where

$$\Gamma = \left[\gamma_{00}, \gamma_{01}, \cdots, \gamma_{0,N+1}, \gamma_{10}, \gamma_{11}, \cdots, \gamma_{1,N+1}, \cdots, \gamma_{N+1,0}, \gamma_{N+1,1}, \cdots, \gamma_{N+1,N+1}\right]^T,$$

and $\tilde{V}(S,t)$ is the approximate solution of the equation (1.3). Then, the following relation holds

$$\begin{aligned} \|V(S,t) - \tilde{V}(S,t)\|^2 &= \iint_D \left[V(S,t) - \tilde{V}(S,t)\right]^2 \mathrm{d}S\mathrm{d}t \\ &= \iint_D \left[V(S,t) - \sum_{i=0}^{N+1}\sum_{j=0}^{N+1} \gamma_{ij} P_i(S) P_j(t)\right]^2 \mathrm{d}S\mathrm{d}t < \varepsilon, \end{aligned}$$

where ε is a very small number.

According to the relation (2.1), the derivatives of the double-barrier option price function can be approximated as follows:

$$\frac{\partial V(S,t)}{\partial S} \simeq \zeta^T(S,t)(M^T \otimes I)\Gamma, \tag{2.2}$$

$$\frac{\partial^2 V(S,t)}{\partial S^2} \simeq \zeta^T(S,t)((M^T)^2 \otimes I)\Gamma, \tag{2.3}$$

$$\frac{\partial V(S,t)}{\partial t} \simeq \zeta^T(S,t)(I \otimes M^T)\Gamma. \tag{2.4}$$

Suppose $\Delta S = \frac{B_u - B_l}{m}$ and $\Delta t = \frac{T}{m}$, are uniform spatial and temporal step lengths, respectively, and $S_i = B_l + i\Delta S$, $i = 0,1,\cdots,m$, and $t_j = j\Delta t$, $j = 0,1,\cdots,m$, are equidistant collocation points, respectively, on the intervals $[B_l, B_u]$ and $[0,T]$. By rewriting the problem (1.3)-(1.4) based on the relations (2.1)-(2.4), and substituting the assumed collocation points, the following system of linear equations is obtained:

$$
\begin{cases}
\zeta^T(S_i,t_j)\Big[(I\otimes M^T)+\frac{1}{2}\delta^2 S_i^{2\beta+2}((M^T)^2\otimes I) \\
\qquad\qquad +(r(t_j)-q(t_j))S_i(M^T\otimes I)-r(t_j)(I\otimes I)\Big]\Gamma=0, \\
\zeta^T(S_i,T)\Gamma=\max(S_i-K,0), \\
\zeta^T(B_l,t_j)\Gamma=\Psi(t_j), \\
\zeta^T(B_u,t_j)\Gamma=\Phi(t_j).
\end{cases}
\tag{2.5}
$$

The system of equations (2.5) can be written in the following matrix form:

$$
A\Gamma=F, \tag{2.6}
$$

where

$$
A=\begin{bmatrix}
\zeta^T(S_i,t_j)\left[(I\otimes M^T)+\frac{1}{2}\delta^2 S_i^{2\beta+2}((M^T)^2\otimes I)+(r(t_j)-q(t_j))S_i(M^T\otimes I)-r(t_j)(I\otimes I)\right] \\
\zeta^T(S_i,T) \\
\zeta^T(B_l,t_j) \\
\zeta^T(B_u,t_j)
\end{bmatrix},
$$

and

$$
F=\begin{bmatrix}
0 \\
\max(S_i-K,0) \\
\Psi(t_j) \\
\Phi(t_j)
\end{bmatrix}.
$$

By solving the system of linear equations (2.6), the unknown coefficients of the Legendre neural network are obtained in such a way that $\|V(S,t)-\tilde{V}(S,t)\|^2$ is minimized. For each internal layer in the system (2.6), to optimize the weight of the hidden layer, the learning machine method Γ uses the least squares approach instead of the process of successive iterations, such that the norm 2 minimizes the output error, that is, the optimal solution of the (2.6) is as $\Gamma^*=\arg\min_\Gamma\|A\Gamma-F\|^2$.

According to the condition of the matrix A, solving the system (2.6) leads to the following three cases:

(1) If A is a square matrix and invertible, then $\Gamma=A^{-1}F$.

(2) If A is rectangular, then $\Gamma^*=A^T(\lambda I+AA^T)^{-1}F$ and Γ^* is the least squares solution of $A\Gamma=F$, i.e. $\Gamma^*=\arg\min_\Gamma\|A\Gamma-F\|^2$. λ is a regularization factor that can be adjusted subject to an specific sample.

(3) If A is a singular matrix, then $\Gamma^*=A^T(\lambda I+AA^T)^{-1}F$.

In this way, the unknown weights of the Legendre neural network are obtained.

Now, we examine the theoretical results presented in the previous section. The values of N and m are considered to be 25 and 100, respectively. The numerical results are extracted from a system with specifications 6500 CPU @ 3.20GHz 3.19GHz-Intel Core(TM) i5 and using the software MATLAB R2015a. The

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double-barrier option pricing problem is solved using the neural network algorithm and considering the following three different cases for the interest rate and dividend parameters [47]:

$$
\begin{aligned}
&1)\ r(t) = 0.075 + 0.05t,\ \ q(t) = 0.05,\\
&2)\ r(t) = 0.1 + 0.05e^{-t},\ \ q(t) = 0.03 + 0.001e^{0.01t},\\
&3)\ r(t) = 0.1 + 0.0005t,\ \ q(t) = 0.02.
\end{aligned}
$$

The expiration date to solve the problem is considered to be in years. The periods selected for solving double barrier option pricing problem are 3 months, 4 months, 6 months, and 1 year. The strike price is 0.5 and the underlying asset price is chosen from the interval $[0, 1]$. The volatility elasticity in the relation $\sigma(S) = \delta S^{\beta}$ takes negative values, because as the underlying asset price increases, $\sigma(S)$ must remain bounded.

Figure 1 shows the double barrier call options price for three different cases of r and q based on the remaining life of the option until the expiration date with a maturity period of 1 year. As can be seen in the figure, at high prices, the longer the remaining life of the option until expiration, the higher the option price. But at low prices, the shorter the remaining life of the option until expiration, the higher the option price.

Figure 2 shows the value of barrier options for $r(t) = 0.075 + 0.05t$ and $q(t) = 0.05$ based on different values of β with 6-month maturity date. Figure 3 shows the value of barrier options for $r(t) = 0.1+0.05e^{-t}$ and $q(t) = 0.03 + 0.001e^{0.01t}$ based on different values of βs with 3-month maturity date.

3. Conclusions

In this study, using the Legendre neural network method based on an improved extreme learning machine algorithm, the evaluation of double-barrier options under the generalized constant elasticity variance model has been investigated. The neural network method is used in the valuation of options to increase the speed of decision-making and improve market forecasts. By learning from historical data, neural networks can predict market movements. Also, neural networks, with the ability to analyze large amounts of historical data in a short time, can identify patterns, complex relationships in the data, and optimal hedging strategies that human traders may overlook. In addition, neural networks enable traders to effectively protect their portfolios against adverse market conditions. In this paper, the risk-free interest rate and dividend parameters of the valuation model are considered to be time-dependent to bring the proposed model closer to the actual market model. The Kronecker product of Legendre polynomials is considered as the basis function of the proposed method. The algorithm procedure is fully described and, while theoretically examining the method, its performance is examined by providing appropriate numerical simulations. The obtained numerical results show the accuracy of the proposed method. Due to the effectiveness of the method investigated, this method can be used for valuation of different types of options and also for calculation of implied volatility.

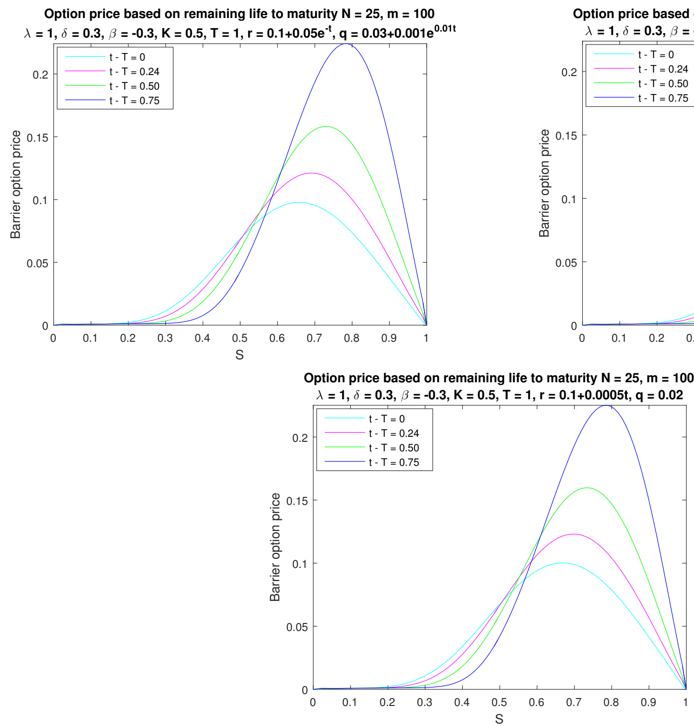

FIGURE 1. Barrier option price for three different cases of r and q based on the remaining life of the option.

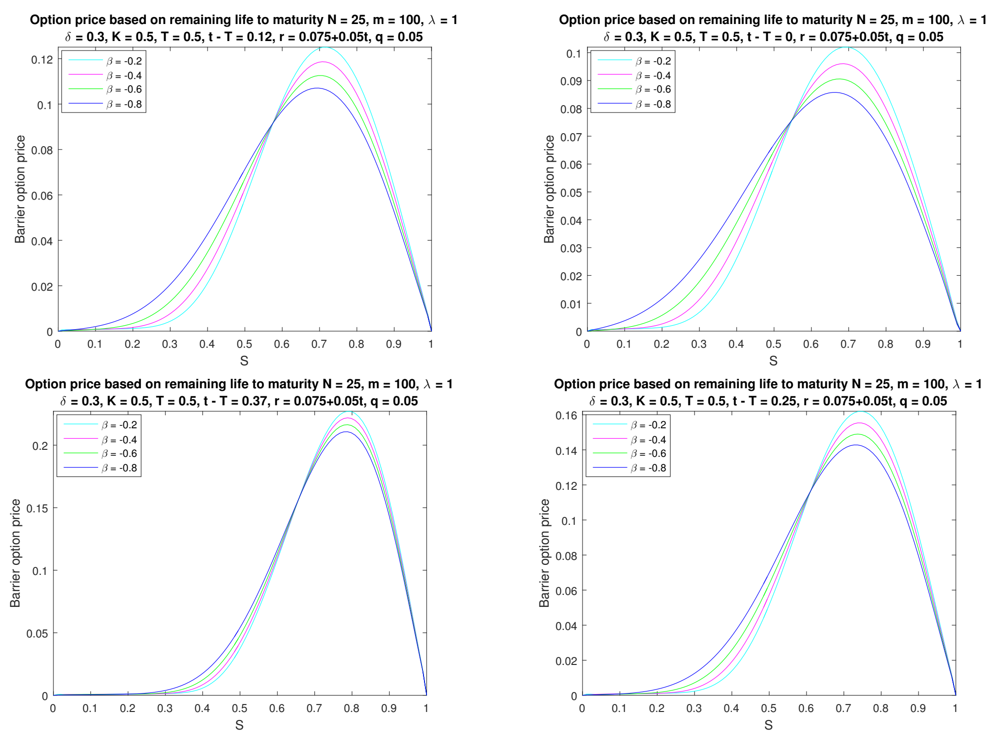

FIGURE 2. Barrier option price for the first case of r and q based on different values of β.

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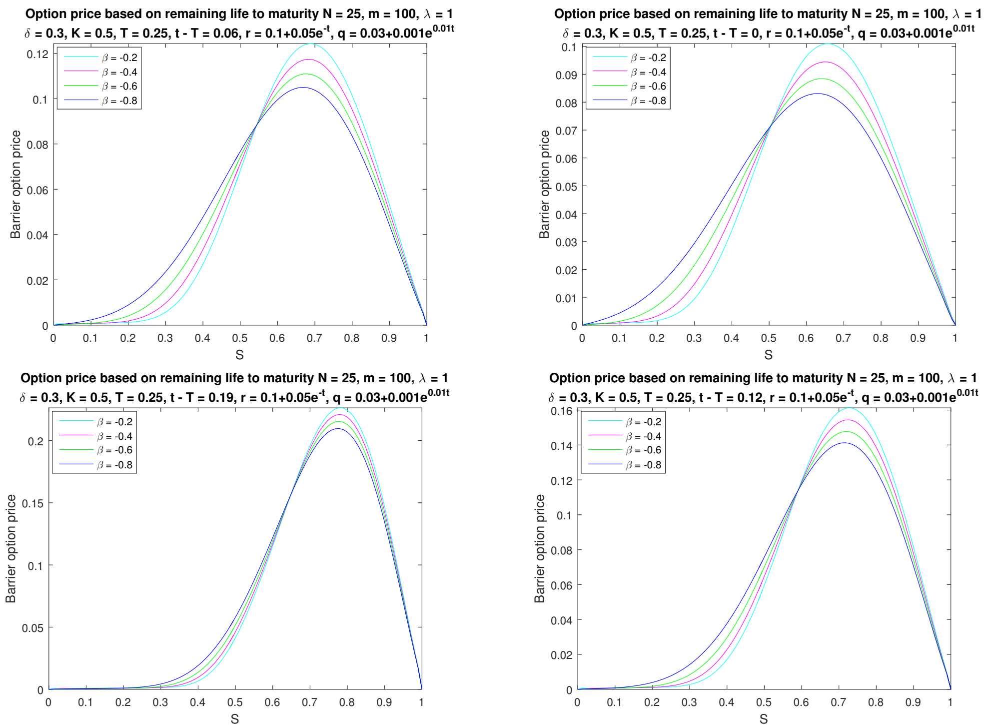

FIGURE 3. Barrier option price for the second case of r and q based on different values of β.

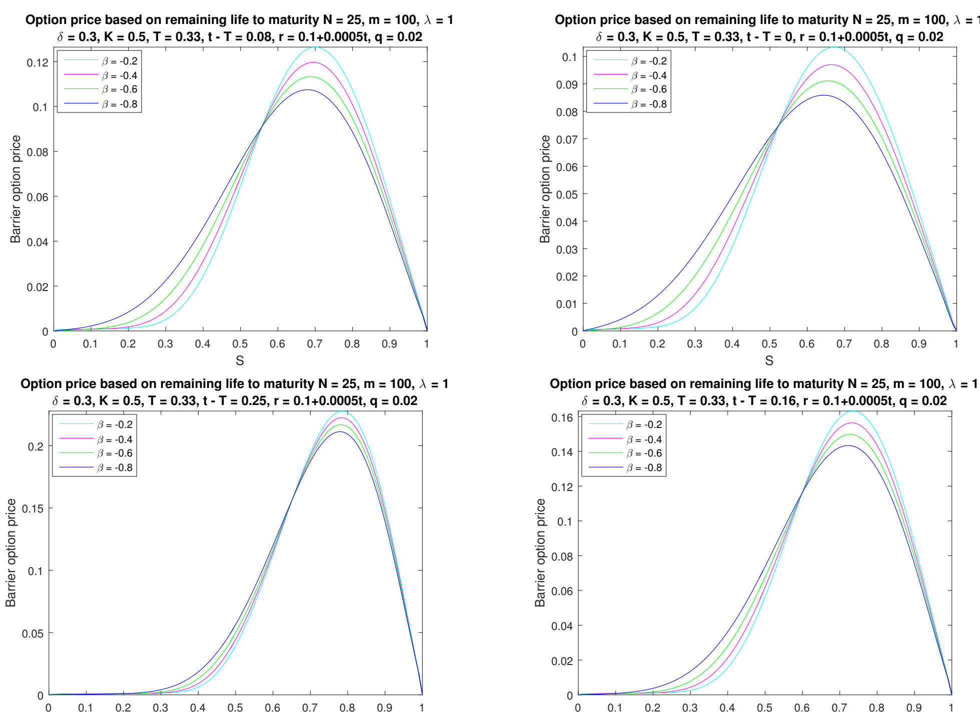

FIGURE 4. Barrier option price for the third case of r and q based on different values of β.

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