

## IDENTIFICATION OF COMMUNITIES IN SOCIAL NETWORKS BASED ON GAME THEORY WITH STABLE COALITIONS

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**ABSTRACT.** Due to the availability of more data and the increase of interactive activities in social media, the identification of overlapping associations has been considered. In this paper, a game theory-based approach to identify overlapping associations is proposed. In this method, association detection is modeled as a coalition formation game. In this game, individuals in a social network are modeled as rational actors whose goal is to improve the group's utility, which is achieved by cooperating with other players and forming coalitions. Each player can join multiple alliances, and alliances with fewer players can merge into a larger alliance as long as the joining operation is conducive to the alliance's goals. Therefore, overlapping associations can be identified simultaneously. In this article, two types of methods based on cooperative and non-cooperative game theory have been discussed. The results report is analyzed based on the comparison of association methods in the form of a diagram. It can be seen that the game group and the COFOGA method perform better association.

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## 1. Introduction

Social network websites provide the possibility of communication between people through the Internet. Networks can have different specifications. One of the most important features is the structure of communities. A community in a network is a group of nodes that are densely interconnected and sparsely connected to other parts of the network. In other words, communities are groups of nodes in a network where nodes within each community are more connected to each other than nodes outside the community. Identifying these communities can help to understand the structural features of social networks and improve user-centricity.

Identification of communities in cases has many applications such as online marketing, determining the location of online advertisements and services such as identifying influential users with the ever-increasing growth in online social networks, and targeted marketing. In the environment of social networks, people's behavior is not completely independent and there is cooperation and interaction between people. We know that game theory is the science of studying human behavior and also modeling the relationship of behavior governing strategic environments. In this environment, each person's behavior and decision-making depends not only on his own behavior, but also on the behavior of others. Therefore, social networks can be analyzed based on game theory. Game theory can be divided into two categories: cooperative games and non-cooperative game [4].

In the model of cooperative games, groups of agents who have come as a team or group to make a profit or win the game, by cooperating with binding agreements and existing rules, they increase the probability of their profit or victory. In a cooperative game collaborative behaviors is studied and agents work together to improve group utility. These groups of agents is called the coalition. Methods based on cooperative game theory consider the formation of an community as a result of group interaction and cooperation. In non-cooperative games, the individual behaviors of the agents are studied and each agent chooses his own strategy to improve his utility. Methods based on non-cooperative game theory consider community formation as the result of individual behaviors of selfish agents and society structure as a balance between individual agents.

In this paper, an almost new approach to identify communities which uses the theory of cooperative and non-cooperative games, is studied.

Identifying communities in social networks is a complex issue, and depending on the characteristics of the network and the purpose of the analysis, there are different methods to identify it [9].

In the article [21, 22] two coalition game models were proposed. The first model focused on the network structure and used the Shapley value to measure people's participation. The second model used both network structure and individual characteristics and assessed participation in connection proximity and preference for specific topics. However, these approaches do not identify overlapping communities.

The first result related to community detection in computer science is probably the graph extraction problem that can be designed VLSI [6] and the modeling of roles and positions in the social structure



should return [18]. While these approaches are relevant to association detection, Newman pointed out several facts that make the approaches unsuitable for association detection. [14]. For example, in a graph extraction problem, the number of nodes to be extracted and the number of groups to be extracted are usually specified in advance, which is very different from our proposed association detection problem. The second serious drawback of these methods based on graph distribution is that they are all basically examples of finding distributions of a graph in such a way that the number of overlapping (interfering) edges between distributions is minimized. However, a small number of overlapping edges may not be a good indication of associations without considering the inherent connections between nodes in the graph.

Newman’s concept of modularity is the first successful attempt to fix the bugs mentioned [14]. Modularity is defined on the superposition of nodes in a graph. Suppose  $G$  is an undirected graph that models a social network with  $n$  nodes and  $m$  edges. Assume that each node  $v$  belongs to the community  $c_v$ . We define the indicator function  $\delta(c_u, c_v) = 1$  if and only if  $c_u = c_v$ , that is, two nodes  $u, v$  are in the same community, otherwise  $\delta(c_u, c_v) = 0$ . The modularity of  $Q$  is the distribution of this special association as follows, where  $A$  is the adjacency matrix of  $G$  with  $A_{uv} = 1$ , when  $uv \in E$  and otherwise  $A_{uv} = 0$ . Also,  $d_u, d_v$  are the degrees of nodes  $u$  and  $v$ , respectively:

$$Q = \frac{1}{2m} \sum_{u,v} (A_{uv} - \frac{d_u d_v}{2m}) \delta(c_u, c_v).$$

## 2. Main results

After introducing the detection of associations based on game theory, some methods introduced in the paper, are experimentally evaluate on real networks and benchmark networks. Two types of methods based on cooperative and non-cooperative game theory have been discussed. The results report is analyzed based on the comparison of association methods in the form of a diagram.

## 3. Conclusions

Social networking websites provide the possibility of communication between people through the Internet. One of the most important features of social networks is the structure of communities in networks. This article has studied a game theory-based approach to identify overlapping communities, which is an important issue in this issue. In the studied method, networks or communities are modeled as a coalition formation game, and individuals in a social network are modeled as rational actors whose goal is to improve the group’s utility, which is achieved by cooperating with other players and forming coalitions. Each player can join multiple alliances, and alliances with fewer players can merge into a larger alliance as long as the joining operation improves the alliance’s goals. Therefore, overlapping communities can be identified simultaneously. Report the results based on the comparison of association methods and it was examined in the form of a diagram. It was observed that the game group and the COFOGA method are better at finding associations.



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