Mathematics and Society
University of Isfahan
http://math-sci.ui.ac.ir

Journal of Mathematics and Society
ISSN (print): 2345-6493, ISSN (on-line): 2345-6507
Vol. 9 No. 2 (2024), pp. 31-47.

http://www.ui.ac.ir

MUSIC AND LINEAR ALGEBRA

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Abstract. Sound, with all its manifestations, presents itself as a form of energy with properties and capabilities that remain partially shrouded in mystery. The art of composing music, the artful assemblage and fusion of sounds, still lingers in a realm of obscurity, fraught with complexities. The techniques wielded by composers often stem from an intrinsic gift that defies replication or sharing. However, akin to other art forms, music can be deciphered through the lenses of mathematics and geometry. Through the utilization of linear algebra, a paramount mathematical discipline, a practical and efficient framework can be established for the comprehension and manipulation of musical compositions. Within this discourse, we delve into the essence of music and its fundamental principles, subsequently exploring the application of mathematical concepts; linear algebra in particular; in the analysis of music. The findings put forth in this manuscript hold the potential to enlighten researchers and enthusiasts within the realm of music and mathematical sciences, enhance comprehension and knowledge, refine and enhance analytical methodologies and musical procedures across various facets of the music industry, and foster a deeper bond between the realms of mathematics and artistic expression.

1. Introduction

Music, regarded as the art of conveying human emotions and inner states of beings, has captivated audiences since ancient times. Over the years, the intertwining of music and mathematics has sparked ongoing curiosity due to its intriguing nature. Drawing on their mathematical acumen and musical

Keywords: Music, melody matrix, note, linear algebra.

Communicated by Saeid Azam.

Article Type: Promotional Paper.

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Received: 18-10-2023, Accepted: 19-02-2024, Published Online: 28-05-2024.

Cite this article: M. Felahat and J. Tayyebi, Music and linear algebra, *Mathematics and Society*, **9** no. 2 (2024) 31–47.

http://dx.doi.org/10.22108/msci.2024.139462.1616 .

flair, composers have masterfully crafted a plethora of musical compositions. The harmonious blend of sounds, intricate patterns, and rhythmic frameworks imbue each genre of music with a unique and individualistic essence. Incorporating elements of linear algebra into music opens up avenues for developing analytical frameworks and fostering creativity in both the creation and reinterpretation of musical pieces. This article delves into the exploration of the correlation between music and linear algebra, aiming to deepen the comprehension of this exquisite art form.

2. **Main Results**

In music, linear transformations and analysis of melody matrices are powerful tools that aid musicians and music analysts in delving into the technical depth of melodies and sound patterns. For example, the Fourier transform allows us to decompose an audio signal into a combination of frequencies, which can be used for analysis and music production. In other words, these transformations enable us to create and examine various patterns of melodies and sounds. On the other hand, melody matrices provide musicians with the opportunity to mathematically model melodies and better understand the connections between notes and musical pieces. By using these matrices, musicians can identify different patterns and rules of melodies and utilize them to create new and inventive musical compositions. As a result, the analysis of melody matrices and linear transformations equip musicians with powerful tools for creating and performing precise and diverse music.

Linear equations in mathematics and linear algebra are powerful tools for describing and analyzing various phenomena and systems. In music, linear equations allow us to determine how a combination of different sounds and frequencies can come together to create a final sound. In composition, a system of linear equations is used to determine the weight and proportion of different sounds within a scale. Generally, to convert a melody into a system of linear equations, we need to represent different parameters of the melody with linear equation variables. By defining these parameters, we can describe musical patterns and structures through the combination of various linear equations. For instance, if we have a simple melody composed of notes A, B, C, and D, we can assign each note to a linear equation variable. For example, use x_1 for note A, x_2 for note B, and so on. By using these defined variables, we can create linear equations. By combining these equations and using linear algebra methods, we can analyze different musical patterns and create a more precise and mathematical expression of musical structures. In music analysis and rhythm prediction, various linear algebra methods such as principal component analysis, independent component analysis, linear regression, etc., are used to extract and identify musical patterns and features. For example, linear regression allows us to examine the relationship between a dependent variable (such as a musical pattern) and several independent variables (like musical features). By utilizing linear equations and matrices, we can analyze the relationships between variables and predict future musical patterns using regression coefficients. Another linear algebra method used for analyzing and predicting musical rhythms is the use of linear differential equations. With these equations, we can investigate temporal changes and

http://dx.doi.org/10.22108/msci.2024.139150.1606

intervals between notes, analyze rhythmic and repetitive patterns. For example, if a musical rhythm involves temporal changes between consecutive notes, we can represent these changes using a set of linear differential equations. These methods help us identify hidden patterns within musical data and gain a deeper understanding of musical structure and rules.

3. Conclusions

This article delves into exploring the fundamental concepts of music and elucidates the relationship between music and linear algebra. The utilization of linear algebra models empowers music analysts to delve more precisely into the internal structures of musical pieces and analyze them. These models are constructed based on geometrical and algebraic concepts, and by maintaining harmonic connections between pieces, they enable us to dissect the structure of music in a more accurate manner. For further reading on this topic, the interested reader is referred to the articles [1, 2, 3].

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