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REAL-WORLD APPLICATIONS OF NUMBER THEORY*

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Abstract. The present paper is concerned with practical applications of the number theory and is intended for all readers interested in applied mathematics. Using examples we show how human creativity can change the results of the pure mathematics into a practical usable form. Some historical notes are also included.

1. Introduction

German mathematician Johann Carl Friedrich Gauss (30 April 1777-23 February 1855), regarded as one of the greatest mathematicians of all time, claimed: *"Mathematics is the queen of the sciences and number theory is the queen of mathematics."* However, for many years number theory had only few practical applications. It is well known that the great English number theorist Godfrey Harold Hardy (7 February 1877-1 December 1947) believed that number theory had no practical applications. See his essay "*A Mathematician's Apology*" [16]. Over the 20th and 21st centuries, this situation has changed significantly. Contrary to Hardy's opinion, many practical and interesting applications of number theory have been discovered. The present paper brings some remarkable examples of number theory applications in the real world. The paper can be regarded as a loose continuation of the author's preceding work [19] and [20].

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Let n be a positive integer, ≥ 2. Then, the equation

$$a_1x_1 + \ldots + a_nx_n = m \tag{1.1}$$

is said to be a linear Diophantine equation if all unknowns $x_1, \ldots, x_n$ and all coefficients $a_1, \ldots, a_n, m$ are integers. For general methods for solving 1.1, see for example [5], [25], and [24, pp. 27-31].

In the following we give some interesting examples of using Diophantine equations in the natural sciences.

2. The results

As the first example we show some applications of a linear Diophantine equation to problems in chemistry. In particular, we will deal with the balancing of chemical equations. See [6]. Consider a chemical equation written in the form

$$x_1A_{a_1}B_{b_1}C_{c_1}\ldots + x_2A_{a_2}B_{b_2}C_{c_3}\cdots + \cdots \to x'_1A'_{a'_1}B'_{b'_1}C'_{c'_1}\cdots + x'_2A'_{a'_2}B'_{b'_2}C'_{c'_3}\cdots + \cdots \tag{2.1}$$

where $A, B, C, \ldots$ are the elements occurring in the reaction, $a_1, b_1, c_1, \ldots, a'_1, b'_1, c'_1, \ldots$ are positive integers or 0, and $x_1, x_2, \ldots, x'_1, x'_2, \ldots$ are the unknown coeffcients of the reactants and products. Then, we hav

$$\begin{aligned} x_1a_1 + x_2a_2 + \cdots &= x'_1a'_1 + x'_2a'_2 + \cdots \\ x_1b_1 + x_2b_2 + \cdots &= x'_1b'_1 + x'_2b'_2 + \cdots \\ x_1c_1 + x_2c_2 + \cdots &= x'_1c'_1 + x'_2c'_2 + \cdots \\ &\cdots \end{aligned} \tag{2.2}$$

Clearly, each equation of (2.2) expresses the law of conservation of the number of atoms for any particular element $A, B, C, \ldots$. Finding all integer solutions $[x_1, x_2, \ldots, x'_1, x'_2, \ldots]$ of (2.2) is a nice elementary problem of Diophantine analysis.

In the second example we show how linear Diophantine equations can be used to determine the molecular formula [6]. Assume that a substance with a molecular weight of m contains elements $A, B, C, \ldots$ with atomic weights $a, b, c, \ldots$ and that $x, y, z, \ldots$ represent the numbers of atoms of $A, B, C, \ldots$ in a molecule. Then, we have

$$ax + by + cz + \ldots = m. \tag{2.3}$$

Let $\alpha, \beta, \gamma, \ldots$ denote the integers nearest the values $a, b, c, \ldots$ and μ denote the integer nearest m. Then, (2.3) can be replaced by the linear Diophantine equation

$$\alpha x + \beta y + \gamma z + \cdots = \mu. \tag{2.4}$$

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If we require that the values $x, y, z, \ldots$ in (2.4) should be reasonably small, we can solve (2.4) under a condition

$$-\frac{1}{2} < (a-\alpha)x + (b-\beta)y + (c-\gamma)z + \cdots < \frac{1}{2}. \tag{2.5}$$

If more solutions of (2.4) are obtained, the true values may be found by substituting into (2.3) and finding which of them satisfies (2.3) with minimum deviation from m.

In the third example we focus on an interesting problem in virology. Recall, that virus particles consist of protein subunits ordered geometrically according to strict symmetry rules. These rules highly depend on the chemical properties of the protein.

As the last example we establish the number $p(n)$ which is the number of partitions of n and it is known under the name of *partitio numerorum*. Two interesting connections between the problem *partitio numerorum* and physics will now be mentioned.

First recall that the Hardy-Ramanujan formula

$$p(n) \sim \frac{1}{4n\sqrt{3}} \cdot \exp(\pi\sqrt{\frac{2n}{3}}) \text{ for } n \to \infty \tag{2.6}$$

has been used, with great success, in quantum physics. The formula (2.6) was discovered in 1917 by G. H. Hardy and the brilliant Indian mathematician Srinivasa Ramanujan (22 December 1887-26 April 1920). The second very important application of Hardy-Ramanujan formula can be found in the problems of statistical mechanics.

3. Conclusions

Finally, some further significant applications of the number theory will be shortly mentioned. Above all, it is well known that the theory of Fibonacci numbers has many applications in physics, chemistry, biology, economy, and architecture. Listing 163 chronological references to papers published from 1611 to 2011, paper [19] can serve as an introduction to this field. Further fields of number theory with important applications include the theory of sequences over finite fields [20]. This theory found an application in the testing of Einstein's general relativity or in testing the global warming of oceans. Furthermore, using methods of elementary number theory, practical problems have been solved concerning to the splicing of telephone cables [21]. Many further interesting applications can be found in the book *Number Theory and the Periodicity of Matter* [3]. Lastly, new attractive applications of the number theory include cryptography, coding theory, and random number generation. With the rise of computers, these fields develop very rapidly with their importance continuously increasing.

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