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INVESTIGATING THE SKILLS OF JUNIOR HIGH SCHOOL STUDENTS IN POSING PROBLEMS IN THE FIELD OF PROPORTIONAL REASONING

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Abstract. This study investigates the skills of junior high school students in posing problems in the field of proportional reasoning. This study, by considering purpose and implementation consecutively, is applied and descriptive (survey type) in nature. The sample of this study was 442 of Qazvin City's junior high school students, who were chosen based on randomized cluster sampling. The measurement tool was a questionnaire with five problem-posing tasks related to proportional reasoning witch its content and face validity were examined by some of the mathematics professors and mathematics education professors. The Cronbach's alpha coefficient for the questionnaire was 0.83. Data analysis was done using SPSS26 software and descriptive and inferential statistics methods. The analysis of the results showed that the student's problem-posing skill was generally evaluated significantly at the level of "replacement" from the theoretical framework of the study. Also, the analysis of data showed that there is a significant difference between the performance of students in 7th, 8th, and 9th grades in problem-posing in the field of proportional reasoning, and with the increasing educational grade, their problem-posing skills will be increased. On the other hand, by studying the effect of students' gender on problem-posing performance, it was found that there was no significant difference between boy and girl students in problem-posing. Also, it was observed that the school type is not an effective factor in problem-posing in proportional reasoning problems and there is no significant difference between the performance of ordinary and gifted school students. The results of this study can be used in teacher Training and textbook authoring.

Keywords: Problem-posing, Proportional reasoning, Students, Junior high school.

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1. Introduction

Proportional reasoning is the cornerstone of high school mathematics and is known as the main goal of the elementary mathematics [1, 2]. Proportional reasoning including ratio, proportion, rate, and fraction are among the important concepts of school mathematics that their learning is necessary for students but it is difficult for teachers to teach [4, 5, 2]. Proportional reasoning is a special mathematical topic in mathematics education research because many subjects need knowledge and understanding in mathematics curriculum (e.g. scale, probability, percentage, rate, calculus, algebra, geometry) [16] and science (density, molarity, speed, force) [17, 13]. Problem-posing is one of the most important aspects of pure and applied mathematics and can be a part of the modeling cycle that is required in modeling real-world phenomena. Based on Leung [43], problem-posing is the new organization of the given problem. In this research, according to the part of the literature review (Vistro-Yu model and Leung model) and evaluation and modification of them, we observed that in the Vistro-Yu model, the incorrect problems were not been considered and in the Leung model, a suitable classifier was proposed for incorrect problem posing in this study, we have proposed a suitable classifier for incorrect classification. Therefore, by choosing the three levels of the five levels of the Leung model that were related to incorrect responses and applying some small changes in it and also levels of Vistro-Yu levels, a combination framework composed of 9 levels was formed and the ability of the junior high school students was studied. In this framework, after reviewing and analyzing project issues, first, the performance of the students is divided into two categories correct problem posing and incorrect problem posing. Incorrect problem posing is classified in three levels and the correct problem posing is classified in six levels. The reason for choosing this combination is that the frameworks used in it were complementary and supported each other, and by combining them, a complete classification was achieved that covers the wide range of students ' problem-posing.

2. Main Results

In this research, five questions related to the concepts need to be provided to the students with different formats and include a specific goal. These five questions were selected as selection criteria and were selected for use in the study to cover all objectives of the study and the extracted data from them were adapted and adapted with the research framework. Therefore, the students' responses to test questions after careful examination and extraction of the results from each of the levels of the proportional problem-posing framework and then the data obtained, led the researchers to the secondary goals of this research, the effect of educational grades, gender, and school type on the performance of students in proportional reasoning. For this purpose, the test questions the target, and the answers provided by students to test questions were analyzed. In general, according to the results of the analysis of students' responses to test questions, the performance of the students in posing proportional

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problems is fairly good. However, most of the students responded to test questions and posed a new problem by changing the variables, numbers, and numerical relationships of the problem and their performance was evaluated mainly at the level of substitution. in addition, a considerable portion of the answers were related to the questions that were posed incorrectly and different factors can have caused this matter such as lack of accuracy to problem assumptions, problem posing with rely on intuition and without reasoning, insufficient understanding of rate concept, incorrect comparison of fractions, misunderstanding in the concept of percent and how to use it and so on. On the other hand, a major part of the incorrect responses is related to the lack of understanding of the nature of the problem, because understanding the nature of a problem is the first step in the solution of the problem [37]. As it was observed a considerable portion of the students with collective strategy and vice versa. After careful examination and analysis, the responses of students to test questions were categorized according to their collective or multiplicative understanding and their proportional reasoning and their proportional reasoning and achieve their goal in each of the nine levels of the theoretical framework of the study. Students' correct responses were classified in levels of 4 to 9, (replacement level to reformatting level) and similarly incorrect responses at levels of 1 to 3, (proposition level to the impossible problem level). The results showed that most of the answers were in level 4 (replacement). In conclusion, according to the distribution of students' responses to levels of the research framework, it can be concluded that the performance of students in posing problems related to proportional reasoning is placed at the replacement level. The outcome of this study is that the percent of the answers to the research questions at the second level (irrelevant problem) can be attributed to the inability of students to distinguish between proportional and non-proportional problems and inappropriate use of proportional reasoning in collective and multiplicative situations, which according to [55, 5, 7] is one of the most important weaknesses of the junior high school students in dealing with proportional reasoning. The root of this issue is the lack of understanding of the existing multiplicative or multiplicative relations between the variables and insufficient understanding of concepts such as ratios, fractions, rates, and proportions. According to the results obtained from the study of the frequency of correct and false responses to research questions, it can be found that 86.1% were correct, indicating that most of the students were able to pose problems related to proportional reasoning. the results of the chi-square test and Spearman correlation coefficient at a 95% confidence level showed that there is a direct and significant relationship between the performance of junior high school students in posing problems related to proportional reasoning and their educational grade with increasing the educational grade of students their problem posing skills in proportional reasoning problems increases too. in general, it makes the development of understanding proportional reasoning abilities. It was also observed that there is no significant relationship between students gender and their performance in posing problems related to proportional reasoning. Despite observing the general differences in the

performance of students in gifted and general schools, this difference is not significant in terms of inferential statistics and chi-square correlation coefficients.

3. Conclusions

According to the results of the analysis of students' responses to the test questions, it is observed that students in response to test and problem posing displayed a variety of functional levels. Some of the students did not understand the concept of the problem, some posed a new problem by only changing the variables, names, numbers, etc., in the problem, and some other added a new variable. Also, it can be noted that the fundamental changes in the problem, contextualizing the problem and making the problem near to the real world, reversing and shifting of demand and data exchanged with each other. Among the causes of low success and low performance in problem posing in proportional reasoning situations, we can mention the lack of accuracy to the problem assumptions, insufficient understanding of rate concept, incorrect comparison of fractions, misunderstanding of the concept of percent and how to use it, etc. The results showed that more than half of the posed problems were in level five (replacement). Although a considerable portion of the responses was in level two (irrelevant problem). Also, educational grade and school type influence the performance of students in posing problems involving proportional reasoning, and at the 95% confidence level, there was a significant difference between students' problem-posing ability and their educational grades. But there was no significant difference between boy and girl students and also students in general and gifted schools.

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