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EDUCATIONAL APPROACHES AND RESEARCH ACHIEVEMENTS OF ALEXANDER KHINTCHINE IN MATHEMATICS

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Abstract. The role of Alexander Yakovlevich Khintchine in mathematics and especially in probability theory is undeniable. His approaches to teaching mathematics, especially in the secondary school, are still a perfect model for mathematics teachers. On the other hand, his role in the theory of infinitely divisible distributions,, distribution of the sum of independent random variables, stable distributions and the domain of attraction of the Gauss law is fundamental and influential. The purpose of this article is to introduce educational approaches and key research achievements in the 30s in mathematics.

1. Introduction

A.Ya. Khintchine was born on July 19, 1894 in the village Kondrovo of the Kaluga region, about one and a half hundred km southwest of Moscow. From 1911 to 1916 he was a student of the Physical-Mathematical faculty of the Moscow State University (MSU). All his scientific life was in deeply connected with this University. In the period of study at the University and in the first years of his research career Khintchine was under a strong influence of the ideas and personality of N.N. Luzin. It is known that A.Ya. Khintchine presented his first result at a meeting of the student mathematical club in November 1914.

Keywords: Education of mathematics, infinitely divisible distributions, stable distributions, limit distribution, the domain of attraction.

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Khintchine was a member of the Soviet delegation at the International Congress of Mathematicians held in Bologna (Italy) from 3 to 10 September 1928. The Russian delegation was represented by 27 scientists including some prominent researchers like S. Bernstein (Karkhov), A.Ya. Khintchine (Moscow), V. Romanovsky (Tashkent) and E. Slutsky (Moscow). We note, however, that Khintchine did not present any communication so that he did not publish a paper in the Proceedings of the Congress (which appeared in 1929-1932).

In connection with the above said motivation for our paper, it is especially important to describe the works by Khintchine because of the following reasons:

- Several important results by Khintchine are forgotten and later rediscovered.
- A number of results were published in inaccessible places and not in English.
- The concrete and clear style of Khintchine's work can help the readers to understand better some recent results.

2. **Main Resalts**

Alexander Yakovlevich Khintchine had a constant and deep interest to the problems of teaching as in universities as in the secondary schools. His pedagogical ideas he has presented in his textbooks, monographs and special articles. In 1938-1940 he headed the physical-mathematical section of the Methodical-educational Soviet at the Ministry of Education of the Russian Federation. When the Academy of Pedagogical Sciences of the Russian Federation was founded, he became an academician of this Academy. He was very active as a member of the editorial board of the multi-volume "Encyclopedy of Elementary Mathematics", some volumes of which appeared in the late 1950s. The first papers by A. Ya. Khintchine were appeared in 1924. In order to understand the role of these articles it is necessary to describe the state of Probability Theory in those years. One can recall a critical review by R. von Mises who summed up of the situation in the following words: To-day, probability theory is not a mathematical science.

The tremendous development of Probability Theory, which thus took place in the twenty years from 1920s to 1940s was, no doubt, a joint effect of the efforts of a number of mathematicians and statisticians. However, it does not seem unlikely that future historians will ascribe its development, as far as the mathematical side of the subject is concerned, above all to the creative powers of four scientists (in alphabetic order): B. de Finetti, A.Ya. Khintchine, A. N. Kolmogorov, and P. Levy. In fact, it may be said that the real turning point came with the publications of the following works:

- P. Levy, Calcul des Probabilites, Gauthier-Villars, Paris, (1925) pp. viii+350.
- B. de Finetti, Funzione caratteristica di un fenomeno aleatorio, *Memorie della R. Accademia Nazionale dei Lincei*, **4** no.5 (1930) 86-133.

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- A. Ya. Khintchine, *Asymptotische Gesetze der Wahrscheinlichkeitsrechnung*, Julius Springer, Berlin, 1933.
- A. N. Kolmogorov, Grundbegriffe der Wahrscheinlichkeitsrechnung, Julius Springer, Berlin, 1933.
- P. Levy, Sur les integrales dont leselements sont des variables aleatoires independentes, *Annali della R. Scuola Normale di Pisa*, **3** (1934) 337-366 and **4** (1935) 217-218.

(1) $B_0 \in T_n(D)$.
(2) $(B_1)_{ij} \in D$ for all $1 \leq i \leq j \leq n-1$.

Let $n > 1$ and $f(X) = B_k X^k + \cdots + B_1 X + B_0 \in \mathrm{Int}(T_n(D))$. Then the following statments hold.

3. **Summary of Proofs**

There was no satisfactory definition of mathematical probability, and the conceptual foundations of the subject were completely obscure. Moreover, with few exceptions, mainly belonging to the French and Russian schools, writers on probability did not seem aware of the standards of rigor which in other mathematical fields, were regarded as obvious. Already in the middle of 1920s were appeared another estimate of Probability Theory as a branch of mathematical science. The Probability Theory has an integral method deeply connected with the methods of modern theory of functions, and thus the most of the recent ideas appeared in the Mathematical Analysis have a fruitful application in the Probability Theory. This optimistic opinion by A. Ya. Khintchine has got an evident justification in the next few decades. At the end of the 1930s, the picture has been radically changed. Mathematical probability theory was firmly established on an axiomatic foundation. It became a purely mathematical discipline, with problems and methods of its own, conforming the current standards of mathematical rigorism, and entering into fruitful relations with other branches of mathematics. At the same time, the fields of applications of mathematical probability were steadily and rapidly growing in number and importance. It is true that nowadays there are still some pure mathematicians who tend to look down on the applied science of probability. But this attitude is expected to disappear within a generation. The tremendous development of Probability Theory, which thus took place in the twenty years from 1920s to 1940s was, no doubt, a joint effect of the efforts of a number of mathematicians and statisticians.

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