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INTRODUCING GEOMETRICAL PARADIGMS AND WORKING SPACES

NARGES YAFTIAN* AND LADAN PAZOKI

Abstract. Despite the many studies conducted in the field of mathematics education and especially geometry, we see that the process of teaching and learning geometry has many challenges. Students' challenges in geometry usually appear when solving problems. Two of the approaches that describe the process of solving geometric problems are called geometrical paradigms and working spaces. The purpose of this article, which is a review article, is to briefly describe these two approaches based on related studies and then show their importance in the process of teaching and learning geometry. In general, geometrical working space refers to the interaction between the three components of visualization, construction and proof. The working space in which a person is reasoning depends on the geometrical paradigm. In Geometry I, the reasoning is based on intuition and experiment, and in Geometry II, the reasoning is made by axioms, but the connection with the physical world is still maintained. Finally, in Geometry III, there's no connection with the physical world and the reasoning is completely logical and abstract. Identifying students' geometrical paradigms and studying the working space in which they solve problems allows teachers to design their teaching according to students' understanding and also in case of difficulties in the process of teaching and learning, they can use a useful approach.

Keywords: Geometry, geometrical paradigms, geometrical working spaces.

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*Corresponding author.

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1. Introduction

Learning geometry causes many challenges for students. Many of these difficulties appear when facing geometric problems and choosing the right strategies for solving or proving them. In general, the way students deal with geometric problems and the strategies they use to solve or prove them, reveals important information about their attitude toward geometry. The geometrical paradigms and working spaces describe the students' way of looking at problems and also the strategies they use in solving them [3, 4].

2. Main Results

Paradigm means all the beliefs held by the members of a community. Learners and teachers with similar paradigms can easily communicate with each other, and when they have different paradigms, many difficulties and misconceptions occur [5]. For example, in the process of proving a claim, it is sometimes allowed to use drawing, but sometimes it is not acceptable and providing more detailed reasonings is needed. Therefore, in different situations and problems, learners use different paradigms and we can't say one paradigm is more accurate than the others. In general, three different geometrical paradigms are introduced by Houdement and Kuzniak [3]. In Geometry I, learners use perception, experiment and connection with the physical world to solve a geometric problem. It is related to reality. So the backward and forward movement between the model and reality is allowed to prove the geometric assertions. In Geometry II, the objects are not material. Definitions and axioms are necessary to define and create the objects, but in this paradigm, they are close to intuition. At last, in Geometry III, the system of axioms has no relation with reality. It is independent of any application of the objects of the real world. This paradigm is mainly used in university courses and it doesn't exist in school geometry. A common educational misconception happens when students and teachers are not in the same paradigm. The passage from one type of Geometry to another is a complex phenomenon. Because it's a change of theory and can be considered an educational evolution. At least, two transitions happen that are not the same. The first transition (from Geometry I to Geometry II) deals with the nature of the objects and the space. The second one (from Geometry II to Geometry III) concerns the system of axioms and it leads to a more complex process. During elementary school, the first transition must happen and teachers can think about how to prepare students for Geometry II [2, 3, 5, 6].

In order to identify the students' geometrical paradigm, it is necessary to examine the strategies they use in problem solving. In fact, the geometrical working space in which they solve problems should be identified. If we consider mathematics as an activity that is done by the human brain, we can find out how learners have a geometrical paradigm. When experts solve geometric problems, they go back and forth between paradigms. A geometrical working space is a place that is organized to explain the

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process of solving geometric problems. It illustrates the structure of the complex situation in which the problem solver acts. It involves two planes which are called the epistemological and the cognitive planes. In the epistemological plane, there are three elements. In fact, learners use three components which are the theoretical system of references, the real space, and artifacts to solve a geometric problem. These components are not sufficient to define the meaning of the geometrical working space clearly. Because it strongly depends on its users too. So the cognitive plane was introduced to describe the cognitive activity of each user which consists of visualization, construction and proof. The process of linking the epistemological plane and the cognitive plane is part of geometrical work. In fact, problem solvers use more than one component to reach the correct response to a problem and a set of these actions represents their geometrical working space [4, 5, 6, 9]. The variety of geometrical working spaces depends on the way users synthesize the cognitive and epistemological planes to solve geometric problems. It also depends on the cognitive abilities of each user too. In fact, being an expert or a beginner in solving problems affects the structure of geometrical working space [4].

3. Summary of Proofs/Conclusion

Many of the difficulties in the process of teaching and learning geometry are due to the difference in the paradigms and working spaces of students and teachers. Several factors influence the students' geometrical paradigm and the working space in which they solve problems, among which the role of the teachers and the textbooks are the most prominent. In fact, the educational system of each country can determine the type of preferred paradigm for each educational level according to the goals of the curriculum. So teachers should be familiar with these approaches and in addition to being aware of them, they can guide the students toward a suitable working space [4, 6].

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Narges Yaftian

Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran.

Email: yaftian@sru.ac.ir

Ladan Pazoki

M.S. in Mathematics Education & Math Teacher in Tehran,, Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran.

Email: pazokiladan@yahoo.com

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